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A new dynamic model of the manual wheelchair for straight and curvilinear propulsion

Félix Chénier, Pascal Bigras, Rachid Aissaoui

***Abstract*— Due to their mechanical design, current wheelchair ergometers cannot simulate the behaviour of a wheelchair propelled on curvilinear paths. This is because they implement a dynamic model of the Wheelchair-user system propelled on Straight Line only (WSL). In this paper, we present a new dynamic model of the Wheelchair-user propelled on Straight and Curvilinear paths (WSC), along with a characterization method based on measurements recorded on the field. Other than measured geometrical constants and kinetic/kinematic data from instrumented wheels, no information about the dynamic parameters such as the system's mass and its moment of inertia are necessary.**

The accuracy of the new WSC model was compared with the WSL model. To this end, ten subjects propelled an instrumented wheelchair following straight and curvilinear patterns. The recorded kinetics were fed to both models, and their estimated kinematics were compared to the recorded ones. For the curvilinear patterns, the RMS relative error between the estimated and measured rear wheels velocities over a complete push cycle are lower for the WSC model than for the WSL model. Outward wheel: 7.98% (WSC) vs 12.98% (WSL). Inward wheel: 10.76% (WSC) vs 20.73% (WSL).

***Index Terms*— Wheelchairs, simulation, dynamics, modeling, ergometers.**

I. Introduction

WHEELCHAIR ergometers are often used to analyze the biomechanics of wheelchair propulsion on level ground. These ergometers aim to replicate the wheelchair behaviour in a controlled laboratory setting. A common setup is to place the rear wheels of the wheelchair on two separate rollers [1], [2], [3], [4], [5], [6]. Most ergometers allow to adjust their inertial and resistive characteristics. Thus, the mass and rolling resistance of the Wheelchair-user system propelled on Straight Line (WSL) can be replicated on the ergometer. However, when the wheelchair is not propelled strictly in a straight line, the vertical moment of inertia of the wheelchair-user system along with the constantly changing caster wheels' orientation are not taken into account by an unidimensional WSL model, despite their high influence on the wheelchair's behaviour [7]. Thus, no actual ergometer can reproduce the propulsion of the wheelchair on a curvilinear path. To that end, a new dynamic model of the wheelchair-user system is necessary.

The WSL model is characterized by measuring the mass of the system and by deducing the rolling resistance from a coast-down test [8]. However, a new model which takes the vertical moment of inertia as a new parameter could not be characterized using this method.

The purpose of this paper is to present a new dynamic model of the Wheelchair-user system propelled on level-ground Straight and Curvilinear paths (WSC). Then, a characterization method based on propulsion data recorded on the field by a wheelchair fitted with instrumented wheels will be presented. The accuracy of the new WSC model will finally be compared to the WSL model when propelling on level ground.

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II. Dynamic models

A. WSL model

As we limit the study to level ground propulsion, the dynamic behaviour of a wheelchair-user system propelled at normal speed on a straight line is given by [9]:

$$m\ddot{x} = \sum_i M_{\text{app}i}/r_R - \text{sgn}(\dot{x})F_{\text{roll}} \tag{1}$$

where m is the mass of the wheelchair-user system, $\dot{x}$ and $\ddot{x}$ are its linear velocity and acceleration, $M_{\text{app}i}$ is the moments applied on the rear wheel i by the user, r_R is the radius of the rear wheels and F_{roll} is the rolling resistance force caused by the deformation of the wheels on the ground.

On a wheelchair ergometer with independent rollers, this equation is implemented by splitting the inertia and resistance in halves onto both rollers. Thus, the WSL model is given by:

$$(m/2)(r_R\ddot{\theta}_{Ri}) = M_{\text{app}i}/r_R - \text{sgn}(\dot{\theta}_{Ri})F_{\text{roll}}/2 \tag{2}$$

where $\dot{\theta}_{Ri}$ and $\ddot{\theta}_{Ri}$ are the angular velocity and angular acceleration of the wheelchair's rear wheel i.

B. WSC model

The development of the new WSC model is based on the following simplifying assumptions:

1) The vertical moment of inertia and the position of the centre of mass of the wheelchair-user system are constant over time.
2) The rolling resistance is constant and laterally equally distributed.

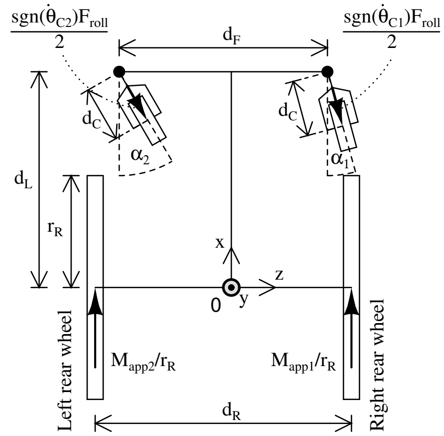


Fig. 1: Dynamic model of the wheelchair-user system.

3) Due to their smaller radius, the caster wheels contribute for a large part of the overall friction [10]. For simplicity, we will assume that all the rolling resistance is mainly due to the caster wheels.
4) The mass and vertical moment of inertia of the caster wheels are negligible compared to the rest of the system; thus, the effect of the caster wheels on the global system is reduced to two resistances acting against the CWOs.

The model will estimate the right and left rear wheel's kinematics as a function of the moments M_{app1} and M_{app2} applied by the user on the rear wheels. According to the geometry of the wheelchair (Fig. 1), the right and left rear wheel's angular accelerations $\ddot{\theta}_{R1}$ and $\ddot{\theta}_{R2}$ can be expressed as a function of the wheelchair's linear acceleration $\ddot{x}$ and its angular acceleration $\ddot{\phi}_y$ around the O reference frame:

$$\ddot{\theta}_{R1,R2} = \ddot{x}(1/r_R) \pm \ddot{\phi}_y(d_R/2r_R) \tag{3}$$

Summing the forces and moments on Fig. 1 yields:

$$m\ddot{x} = (M_{\text{app1}} + M_{\text{app2}})/r_R - \frac{F_{\text{roll}}}{2}\Big(\text{sgn}(\dot{\theta}_{C1})\cos(\alpha_1) + \text{sgn}(\dot{\theta}_{C2})\cos(\alpha_2)\Big) \tag{4}$$

$$\begin{aligned} I_{0y}\ddot{\phi}_y = {} & \frac{d_{\text{R}}}{2r_R}(M_{\text{app1}} - M_{\text{app2}}) \\ & + \frac{d_F F_{\text{roll}}}{4}\Big(\text{sgn}(\dot{\theta}_{C2})\cos(\alpha_2) - \text{sgn}(\dot{\theta}_{C1})\cos(\alpha_1)\Big) \\ & - \frac{d_L F_{\text{roll}}}{2}\Big(\text{sgn}(\dot{\theta}_{C2})\sin(\alpha_2) + \text{sgn}(\dot{\theta}_{C1})\sin(\alpha_1)\Big) \end{aligned} \tag{5}$$

where m is the total mass of the wheelchair-user system, I_{0y} is the moment of inertia of the system around the O reference frame, α_1 and α_2 are the caster wheels' orientations, $\dot{\theta}_{C1}$ and $\dot{\theta}_{C2}$ are the caster wheels' angular velocities, and d_R, r_R, d_F and d_L are geometrical constants. The variables α_1, α_2 as well as $\dot{\theta}_{C1}$ and $\dot{\theta}_{C2}$ are predicted from the kinematic model of a wheelchair, based on the rear wheels' velocities [11]:

$$\dot{\alpha}_i = \frac{r_R(\dot{\theta}_{R1} - \dot{\theta}_{R2})}{d_C d_R}\left(d_L\cos\alpha_i \mp \frac{d_F\sin\alpha_i}{2} - d_C\right) - \frac{r_R(\dot{\theta}_{R1} + \dot{\theta}_{R2})}{2d_C}\sin\alpha_i \text{ for } i \in \{1,2\} \tag{6}$$

where $\dot{\theta}_{R1}$ and $\dot{\theta}_{R2}$ are the angular velocities of the right and left rear wheels and are obtained by a time integration of (3).

The rolling direction of the caster wheels is expressed by [11]:

$$\text{sgn}(\dot{\theta}_{Ci}) = \text{sgn}\{\frac{r_R(\dot{\theta}_{R1} + \dot{\theta}_{R2})}{2}\cos\alpha_i + \frac{r_R(\dot{\theta}_{R1} - \dot{\theta}_{R2})}{d_R}(d_L\sin\alpha_i \pm (d_F/2)\cos\alpha_i)\} \text{ for } i \in \{1,2\} \tag{7}$$

III. Characterization

The model must be characterized to fit the propulsion dynamics of the subject. The geometric constants d_F, d_L, d_C and r_R are easily measured, but the dynamic constants m, I_{0y} and F_{roll} are more difficult to measure experimentally. They will be estimated using a least-square regression approach.

Equations (4) and (5) can be rewritten to isolate the dynamic parameters m, I_{0y} and F_{roll}. We obtain the following system:

$$\mathbf{m}_{\text{app}} = \mathbf{W}\mathbf{a}$$

where $\mathbf{m}_{\text{app}}$ is the input vector, $\mathbf{W}$ is the state matrix and $\mathbf{a}$ is the unknown parameters vector.

$$\mathbf{m}_{\text{app}} = \begin{bmatrix} M_{\text{app1}} \\ M_{\text{app2}} \end{bmatrix} \quad \mathbf{a} = \begin{bmatrix} m \\ I_{0y} \\ F_{\text{roll}} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} \frac{r_R\ddot{x}}{2} & \frac{r_R\ddot{\phi}_y}{d_R} & \frac{r_R}{2}\left(\frac{c_1+c_2}{2} + \frac{d_F(c_1-c_2)}{2d_R} + \frac{d_L(s_1+s_2)}{d_R}\right) \\ \frac{r_R\ddot{x}}{2} & \frac{-r_R\ddot{\phi}_y}{d_R} & \frac{r_R}{2}\left(\frac{c_2+c_1}{2} + \frac{d_F(c_2-c_1)}{2d_R} - \frac{d_L(s_2+s_1)}{d_R}\right) \end{bmatrix}$$

where $c_i = \text{sgn}(\dot{\theta}_{Ci})\cos\alpha_i$ and $s_i = \text{sgn}(\dot{\theta}_{Ci})\sin\alpha_i$.

Knowing $\mathbf{m}_{\text{app}}$ and $\mathbf{W}$, a least-square regression could be used to estimate $\mathbf{a}$. However, whereas $\mathbf{m}_{\text{app}}$ is obtained directly using the moments recorded by the instrumented wheels, $\mathbf{W}$ is not directly measurable and will need to be estimated.

Both instrumented rear wheels have a built-in position encoder; it will be used to estimate the linear and angular accelerations of the wheelchair. However, as the values returned by the encoders are quantized, their double-derivatives are prone to errors due to the quantification noise. A solution is to filter the quantized angular position by a double-derivative low-pass filter. From the geometry of the wheelchair, we get:

$$\left(g(t) * \hat{\dot{x}}\right) = \frac{r_R}{2}\left(\ddot{g}(t) * (\hat{\theta}_{R1} + \hat{\theta}_{R2})\right)$$

$$\left(g(t) * \hat{\dot{\phi}}_y\right) = \frac{r_R}{d_R}\left(\ddot{g}(t) * (\hat{\theta}_{R1} - \hat{\theta}_{R2})\right)$$

where $\hat{\theta}_{R1}$ and $\hat{\theta}_{R2}$ are the quantized angular positions of the right and left rear wheels, $g(t)$ is the impulse-response of the low-pass filter, $\ddot{g}(t)$ denotes the double-derivative and the $*$ symbol is the convolution operator.

The caster wheels' orientations can be estimated $(\hat{\alpha}_1, \hat{\alpha}_2)$ based on the rear wheels' kinematics [11], using (6). To keep the phase and attenuation caused by $g(t)$ uniform, $\mathbf{m}_{\text{app}}$ and all the content of $\mathbf{W}$ must be equally filtered by $g(t)$. Thus, instead of solving $\mathbf{m}_{\text{app}} = \mathbf{W}\mathbf{a}$, a least-square regression will be applied on:

$$g(t) * \mathbf{m}_{\text{app}} = (g(t) * \hat{\mathbf{W}})\mathbf{a}$$

where:

$$(g(t) * \hat{\mathbf{W}}) = \big[[GW1][GW2]\big]$$

$$[GW1] = \begin{bmatrix} \frac{r_R(g(t)*\hat{\dot{x}})}{2} & \frac{r_R(g(t)*\hat{\dot{\phi}}_y)}{d_R} \\ \frac{r_R(g(t)*\hat{\dot{x}})}{2} & \frac{-r_R(g(t)*\hat{\dot{\phi}}_y)}{d_R} \end{bmatrix}$$

$$[GW2] = \begin{bmatrix} \frac{r_R}{2} g(t) * \left(\frac{\hat{c}_1+\hat{c}_2}{2} + \frac{d_F(\hat{c}_1-\hat{c}_2)}{2d_R} + \frac{d_L(\hat{s}_1+\hat{s}_2)}{d_R}\right) \\ \frac{r_R}{2} g(t) * \left(\frac{\hat{c}_2+\hat{c}_1}{2} + \frac{d_F(\hat{c}_2-\hat{c}_1)}{2d_R} - \frac{d_L(\hat{s}_2+\hat{s}_1)}{d_R}\right) \end{bmatrix}$$

for $\hat{c}_i = \left(\text{sgn}(\hat{\dot{\theta}}_{Ci}) \cos \hat{\alpha}_i\right)$ and $\hat{s}_i = \left(\text{sgn}(\hat{\dot{\theta}}_{Ci}) \sin \hat{\alpha}_i\right)$.

IV. Experimental method

Ten healthy subjects participated in this study. The experimental method was approved by the ethics committee of the École de technologie supérieure (ÉTS), Montréal, Canada. Data acquisitions were done at ÉTS in a large empty room. The floor was made of flat vinyl tiles. All subjects used the same Invacare Ultralight A4 wheelchair instrumented with two SmartWeels (Three Rivers Holdings, LLC). The weights of the subjects and of the wheelchair were measured using an AMTI OR6-7 1000 force platform (Advanced Mechanical Technology, Inc.) The angular position of the rear wheels along with the moments applied by the subjects were simultaneously measured at 240Hz.

A. Characterization

The following sequences were executed for each subject:

Sequence 1: After starting and synchronizing the instruments, the subject applied one synchronous bilateral push on the rear wheels from a static position with the caster wheels trailing behind, and then waited for the wheelchair to stop by itself (due to friction only). The same trial was repeated 10 times.

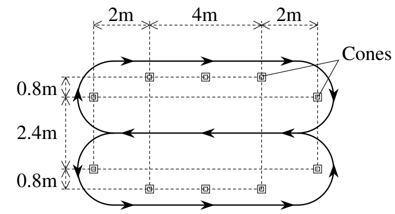


Fig. 2: Controlled path for the straight and curvilinear continuous propulsion

Sequence 2: Again from a static position with the caster wheels trailing behind, the subject applied one unilateral push on one wheel only, and waited for the wheelchair to stop by itself. The same trial was repeated 10 times with alternating hands.

For the WSL model, the wheelchair's and user's masses were measured, whereas the rolling resistance was estimated by performing a coast-down [12], using $F_{\text{roll}} = -(G(t) * \ddot{x})$ based on straight line deceleration data from sequence 1.

For the WSC model, the three parameters m, I_{0y} and F_{roll} were obtained by the characterization method presented in section III, based on data from sequences 1 and 2.

B. Validation

To test both models for straight and curvilinear propulsion, each subject propelled the wheelchair following the controlled path of Fig. 2. The path contained three 6m straight line sections and four 1m-radius U-turns in both directions. The wheeling speed was controlled at around 1m/s, which is a common natural wheeling speed [13], by measuring the time needed to complete a lap.

The WSL and WSC models were implemented in Matlab/Simulink (Mathworks, Inc.). After both models were characterized, the rear wheels velocities were estimated by both models based on the recorded kinetics from the continuous propulsion. The accuracy of the models was obtained by comparing the estimated rear wheels velocities to the measured rear wheel velocities. However, as both models use integrators to estimate the velocities, their outputs are prone to drift since the modelling errors are integrated. As an illustration of this drift, Fig. 3 shows an excerpt of the results for which the drift has not been corrected. To correct this drift, the estimated velocity errors were reset on each contact between the hands and the hand rims. For each propulsion cycle, the RMS velocity error was calculated on a complete push cycle, i.e. from the first hand(s) contact with the hand rim(s) to the next one.

V. Results

A. Characterization

Table I shows the predicted and measured parameters of the WSC model, for each subject. The parameters of the WSL model are shown in Table II.

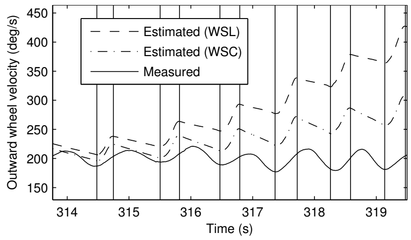


Fig. 3: Excerpt of the measured and estimated velocities of the outward wheel on a typical curvilinear continuous propulsion, when the drift is not corrected. Vertical bars denote the starts and ends of the push phases.

TABLE I: Parameters of the WSC model.

Subject	Param	Value	Error	Unit
1	m	110.81	13.83	kg
	I_0	8.01	-	kg·m^2
	F_{roll}	14.92	-	N
2	m	114.78	5.30	kg
	I_0	9.73	-	kg·m^2
	F_{roll}	18.62	-	N
3	m	90.28	-0.90	kg
	I_0	8.25	-	kg·m^2
	F_{roll}	14.72	-	N
4	m	82.29	-3.09	kg
	I_0	6.30	-	kg·m^2
	F_{roll}	12.96	-	N
5	m	98.49	-3.99	kg
	I_0	5.86	-	kg·m^2
	F_{roll}	12.04	-	N
6	m	94.74	7.86	kg
	I_0	6.28	-	kg·m^2
	F_{roll}	13.46	-	N
7	m	94.61	4.33	kg
	I_0	6.82	-	kg·m^2
	F_{roll}	11.43	-	N
8	m	85.63	6.45	kg
	I_0	6.71	-	kg·m^2
	F_{roll}	10.89	-	N
9	m	106.46	-3.12	kg
	I_0	7.40	-	kg·m^2
	F_{roll}	17.45	-	N
10	m	109.93	10.55	kg
	I_0	7.93	-	kg·m^2
	F_{roll}	14.60	-	N

TABLE II: Parameters of the WSL model.

Subj.	m (kg)	F_{roll} (N)
1	97.0	13.37
2	109.5	16.84
3	91.2	12.42
4	85.4	10.24
5	102.5	13.99
6	86.9	12.28
7	90.3	12.28
8	79.2	9.90
9	109.6	14.77
10	99.4	13.57

TABLE III: Mean and standard deviation of the RMS absolute error on the estimated wheel velocities in continuous propulsion (deg/s, mean $\pm$ st.dev.).

Category, wheel	WSL model	WSC model
Straight line, left wheel	12.66 ± 2.10	12.79 ± 3.37
Straight line, right wheel	13.06 ± 3.04	12.30 ± 2.46
U-turn, pushed (outward) wheel	22.75 ± 4.69	22.95 ± 6.08
U-turn, opposite (inward) wheel	15.55 ± 4.26	12.55 ± 4.53

TABLE IV: Mean and standard deviation of the RMS relative error on the estimated wheel velocities in continuous propulsion (%, mean $\pm$ st.dev.).

Category, wheel	WSL model	WSC model
Straight line, left wheel	6.04 ± 1.58	5.95 ± 1.77
Straight line, right wheel	6.06 ± 1.49	5.67 ± 1.31
U-turn, pushed (outward) wheel	12.98 ± 2.86	7.98 ± 1.87
U-turn, opposite (inward) wheel	20.73 ± 6.48	10.76 ± 3.77

B. Validation

Data was separated into two categories: bilateral propulsion in straight line and unilateral propulsion in the U-turns parts of the path. For each subject and category, 10 push cycles were selected for the analysis. The mean RMS error for those 10 propulsion cycles are shown in Fig. 4 and resumed in Table III. The estimation error is also expressed relatively to the real velocities in Table IV. We observe that the estimation accuracy is similar for both models when the wheelchair is propelled on straight line. However, when the wheelchair is propelled on curvilinear paths, the estimation error of the WSC model is cut by half compared to the the WSL model.

VI. Discussion

Table I shows that the estimated masses are similar to their real values, with a mean $\pm$ st.d. error of 3.72 ± 6.24kg. Additionally, the estimated rolling resistances are similar between the new characterization process and the conventional coast-down method (Table II), with a mean $\pm$ st.d. difference of 1.14 ± 1.50N. However, whereas those results are highly encouraging, we cannot conclude on the estimation accuracy of the moment of inertia because no reference value is available.

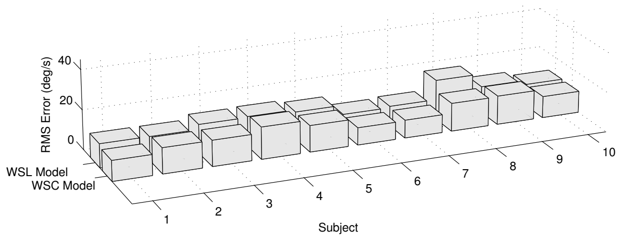
Fig. 4a and Tables III and IV suggest that there is little difference between the WSL and WSC models for continuous synchronous propulsion in straight line. This can be explained by evaluating (4) for the wheelchair moving in straight line. First, the caster wheels are aligned with the wheelchair frame:

$$\alpha_1 = \alpha_2 = 0^\circ$$

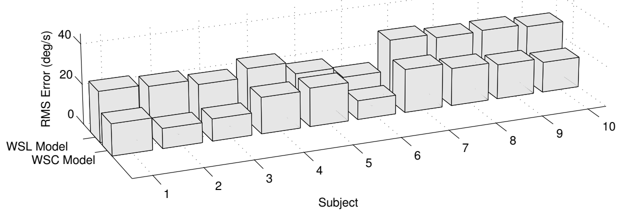
Second, the caster wheels are both rolling in the same direction than the wheelchair:

$$\text{sign}(\dot{\theta}_{C1}) = \text{sign}(\dot{\theta}_{C2}) = \text{sign}(\dot{x})$$

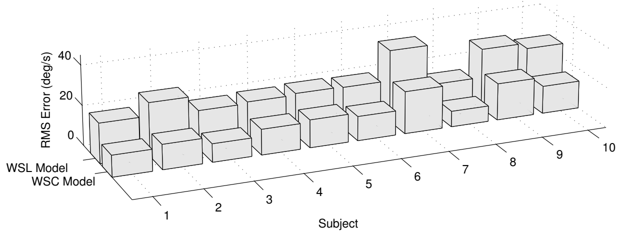
From those simplifications, (4) resolves to (1), which is the dynamic equation of the WSL model. Therefore, both



(a) Right wheel velocity error when performing straight line bilateral pushes. (Left wheel results are similar.)



(b) Pushed (outward) wheel velocity error when performing unilateral pushes during a 1m-radius U-turn.



(c) Opposite (inward) wheel velocity error when performing unilateral pushes during a 1m-radius U-turn.

Fig. 4: RMS Error of the estimated wheel velocities in continuous propulsion.

dynamic models are equivalent for straight line propulsion. However, for unilateral propulsion during a U-turn, the estimated velocities are more accurate for the WSC model, for both the outward wheel (Fig. 4b) and the inward wheel (Fig. 4c).

Finally, Tables III and IV still shows a significant error between the estimated and real velocities. This is mostly due to modelling errors caused by the simplifying hypothesis. For example, the first hypothesis stated that the vertical moment of inertia and the position of the centre of mass of the wheelchair-user system are constant over time. We know that this is not really the case because of the fore-and-aft movement of the subject on the wheelchair. As an effect, this movement causes an acceleration of the system during the beginning of the recovery phase [14], which is visible on Fig. 3. This effect is not replicated by any of the presented models: to be replicated, a model of the human body along with kinematic recordings would need to be included in the dynamic model of the wheelchair-user system.

VII. Conclusion

A new dynamic model of the wheelchair-user system was developed. It includes the dynamic effects of the system's vertical moment of inertia and the orientation of the caster wheels. A characterization procedure was also established, based on a least-square regression of the dynamic parameters. This regression is performed from propulsion data collected on the field by instrumented wheels, when the user is executing simple manoeuvres. As a validation of the WSC model, we recorded the kinetics and kinematics from a wheelchair instrumented with two SmartWheels for ten subjects. We then compared the estimated kinematics from the WSL and WSC models to the real measured kinematics. For the straight patterns, the WSC wheelchair-user model is equivalent to the WSL model. However, for curvilinear patterns, the WSC model is more accurate, with a RMS velocity error on a whole propulsion cycle of: 7.98% (WSC) vs 12.98% (WSL) for the outward wheel, and 10.76% (WSC) vs 20.73% (WSL) for the inward wheel.

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